GCE AS/A level

0978/01

MATHEMATICS – FP2
Further Pure Mathematics

A.M. TUESDAY, 18 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Using the substitution $u = x^2$, evaluate the integral

$$\int_1^2 \frac{x}{\sqrt{25 - x^4}}\,\mathrm{d}x.$$

Give your answer correct to three significant figures. [5]

2. Consider the equation

$$\sin\theta + 3\cos\theta = 2.$$

(a) Putting $t = \tan\left(\frac{\theta}{2}\right)$, show that

$$5t^2 - 2t - 1 = 0.$$ [3]

(b) Hence find the general solution of the above trigonometric equation, giving your answers in radians. [6]

3. *(a)* Find the four fourth roots of -1, giving your answers in the form $x + \mathrm{i}y$. [6]

(b) (i) Plot the points corresponding to these roots on an Argand diagram.

(ii) The points are joined up to form a square. Find the area of the square. [3]

4. The function f is defined on the domain $x > 1$ by

$$f(x) = \frac{2x + 3}{x - 1}.$$

(a) Show that f is a strictly decreasing function. [3]

(b) Given that $S = [4, 5]$, determine

(i) $f(S)$,

(ii) $f^{-1}(S)$. [6]

5. The ellipse E has equation

$$x^2 + 2y^2 - 4x + 4y + 2 = 0.$$

(a) Find

(i) the coordinates of the centre,

(ii) the eccentricity,

(iii) the coordinates of the foci,

(iv) the equations of the directrices. [9]

(b) (i) Show that the y-axis is a tangent to E.

(ii) Find the gradient of the tangent, other than the y-axis, from the origin to E. [7]

6. *(a)* Express

$$\frac{4x^2 - 2x + 9}{x\left(x^2 + 3\right)}$$

in partial fractions. [4]

(b) Hence evaluate

$$\int_1^3 \frac{4x^2 - 2x + 9}{x\left(x^2 + 3\right)}\,\mathrm{d}x,$$

giving your answer correct to three significant figures. [6]

7. The function f is defined by

$$f(x) = \frac{\left(2x^2 + 1\right)^2}{x^3}.$$

(a) Determine whether f is even, odd or neither even nor odd. [3]

(b) Find the x-coordinates of the stationary points on the graph of f. [4]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f and its asymptotes. [2]

8. Using de Moivre's Theorem, show that

$$\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta,$$

where a, b, c are constants whose values are to be determined. [6]

